Understanding FFT Windows

Introduction

FFT based measurements are subject to errors from an effect known as leakage. This effect occurs when the FFT is computed from of a block of data which is not periodic. To correct this problem appropriate windowing functions must be applied. The user must choose the appropriate window function for the specific application. When windowing is not applied correctly, then errors may be introduced in the FFT amplitude, frequency or overall shape of the spectrum. This application note describes the phenomenon of leakage, the various windowing functions and their strengths and weaknesses, and examples are given for various applications.

FFT Background

Most dynamic signal analyzers (Figure 1) compute time and frequency measurements. Time measurements include capturing time traces of measured sig-

Figure 1. Dactron Focus™ (left) and the Dactron Photon™ (right) Dynamic Signal Analyzers

nals, including filtering and statistical measures. Frequency measurements that are computed by most DSAs include Fast Fourier Transform, Power Spectral Density, Frequency Response Functions, Coherence and many more. These signals are computed in the DSP from the digitized time data. Time data is digitized and sampled into the DSP block by block. A block is a fixed number of data points in the digital time record. Most frequency functions are computed from one block of data at a time. A block of data is also called a time record or time window.

The Fast Fourier Transform (FFT) is the Fourier Transform of a block of time data points. It represents the frequency composition of the time signal. Figure 2 shows a 10 Hz sine waveform (top) and the FFT of the sine waveform (bottom). A sine wave is composed of one pure tone indicated by the single discrete peak in the FFT with height of 1.0 at 10 Hz.
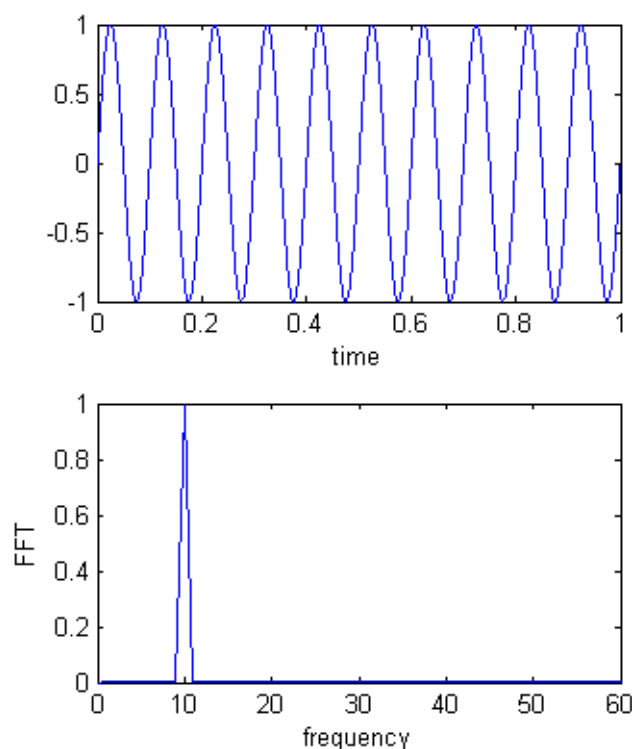

Figure 2. Time waveform of sine function (top) and FFT (bottom).

Leakage

The FFT computation assumes that a signal is periodic in each data block, that is, it repeats over and over again and it is identical every time. Note this was the case in Figure 2 because there are an interger number of cycles of the sine wave in the data record. Another type of signal that satisfies the periodic requirement is a transient signal that starts at zero at the beginning of the time window and then rises to some maximum and decays again to zero before the end of the time window.

When the FFT of a non-periodic signal is computed then the resulting frequency spectrum suffers from leakage. Leakage results in the signal energy smearing out over a wide frequency range in the FFT when it should be in a narrow frequency range. Figure 3 illustrates the effect of leakage. The left-top graph shows a 10 Hz sine wave with amplitude 1.0 that is periodic in the time frame. The resulting FFT (bottom-left) shows a narrow peak at 10 Hz in the frequency axis with a height of 1.0 as expected. Note the dB scale is used to highlight the shape of the FFT at low levels. The right-top graph shows a sine wave that is not periodic in the time frame resulting in leakage in the FFT (bottom-right). The amplitude is less than the expected 1.0 value and the signal energy is more dispersed. The dispersed shape of the FFT makes it more difficult to identify the frequency content of the measured signal.
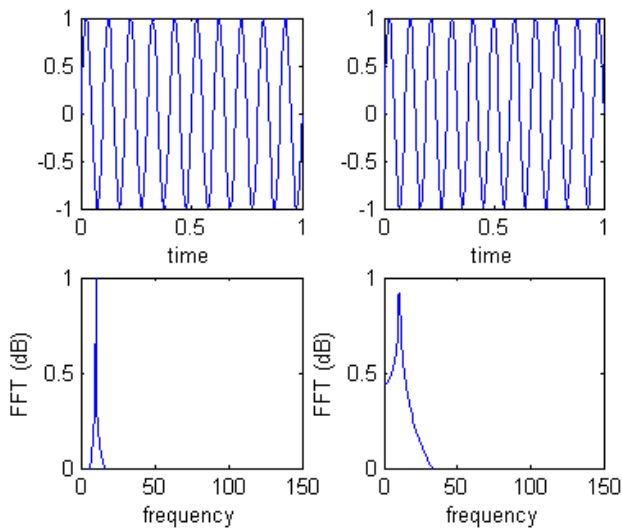

Figure 3. Comparison of periodic sine wave (left) and FFT to non periodic (right) with leakage in the FFT.

Windowing Reduces Leakage

In a signal analyzer the time record length is adjustable but it must be selected from a set of predefined values. Since most signals are not periodic in the predefined data block time periods, a window must be applied to correct for leakage. A window is shaped so that it is exactly zero at the beginning and end of the data block and has some special shape in between. This function is then multiplied with the time data block forcing the signal to be periodic. A special weighting factor must also be applied so that the correct FFT signal amplitude level is recovered after the windowing. Figure 4 shows the effect of applying a Hanning window to a pure sine tone. The left-top plot shows a sine tone that is not periodic in the time window without the windowing function resulting in leakage in the FFT (left-bottom).

When a Hanning window is applied (top-right), then the leakage is reduced in the FFT (bottom-right). The resulting spectrum is a sharp narrow peak with amplitude of 1.0. Notice that it does not have exactly the same shape as the FFT of the original periodic sine wave in Figure 3, but the amplitude and frequency errors resulting from leakage are corrected. A Windowing function minimizes the effect of leakage to better represent the frequency spectrum of the data.
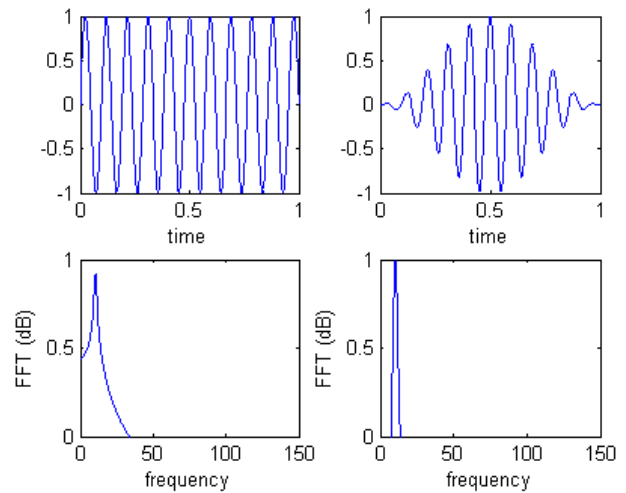

Figure 4. Comparison of non periodic sine wave and FFT with leakage (left) to windowed sine wave and FFT showing no leakage (right).

Windowing functions are most easily understood in the time domain; however, they are often implemented in the frequency domain instead. Mathematically there is no difference when the windowing is implemented in the frequency or time domains, though the mathematical procedure is somewhat different. When the window is implemented in the frequency domain, the FFT of the window function is computed one time and saved in memory and then it is applied to every FFT frequency value correcting the leakage in the FFT. This gives rise to one measure of the window's characteristics, known as the side lobe. The FFT of a window has a peak at the applied frequency and other peaks, called side lobes, on either side of the applied frequency. The height of the side lobes indicates what affect the windowing function will have on frequencies around the applied frequency. In general, lower side lobes reduce the leakage in the measured FFT but increase the bandwidth of the major lobe.

Figure 5 shows the Hanning windowing function and its FFT. The highest side lobe is -32 dB. Compare this with the Flat Top windowing function in Figure 6. The highest side lobe is much lower (-74), but the main lobe bandwidth is significantly wider.
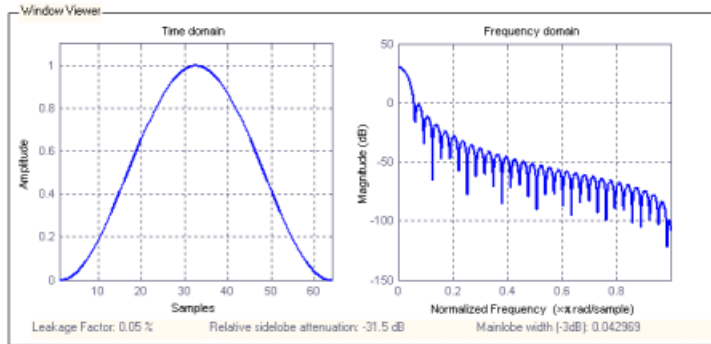

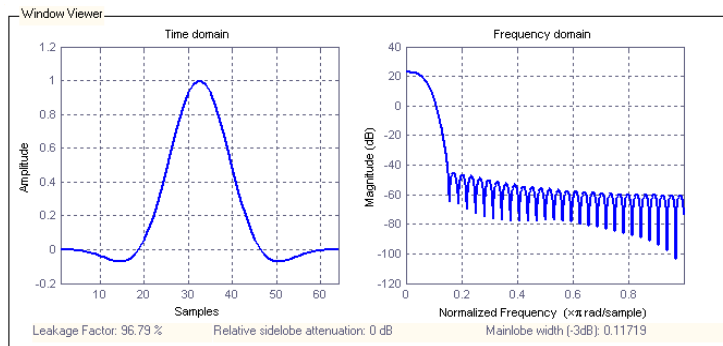*Figure 5. Hanning window (left) and FFT (right).*

Figure 6. Flat Top window (left) and FFT (right).

A comparison of an FFT of a non-periodic sine wave with Hanning and Flat Top windows is shown in Figure 7.
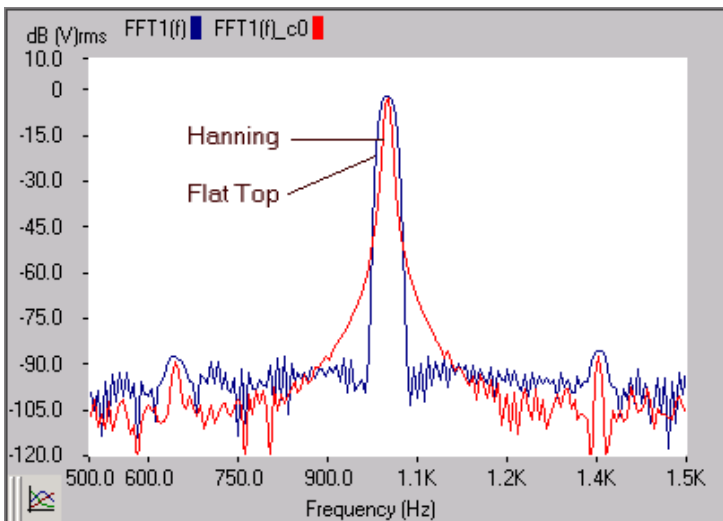

Figure 7. Comparison of Hanning and Flat Top windowing functions with a non-periodic sine wave.

Figure 8 shows a frequency response function of a beam measured with an impact hammer and accelerometer with and without a window. In this case, leakage drastically affects the overall shape of the spectrum. The unwindowed spectrum totally obscures the first anti-resonance and it also caused some aplitude errors in the spectrum peaks that correspond to the structure's resonances.
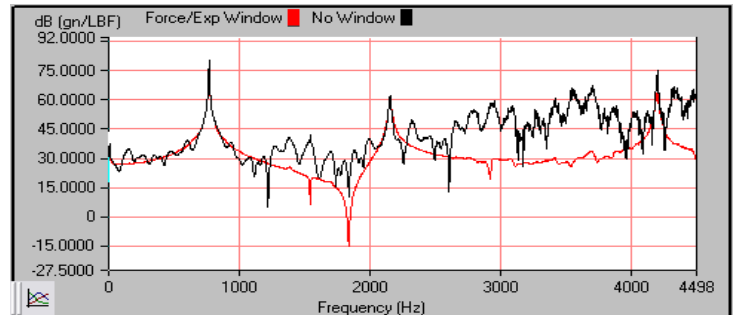

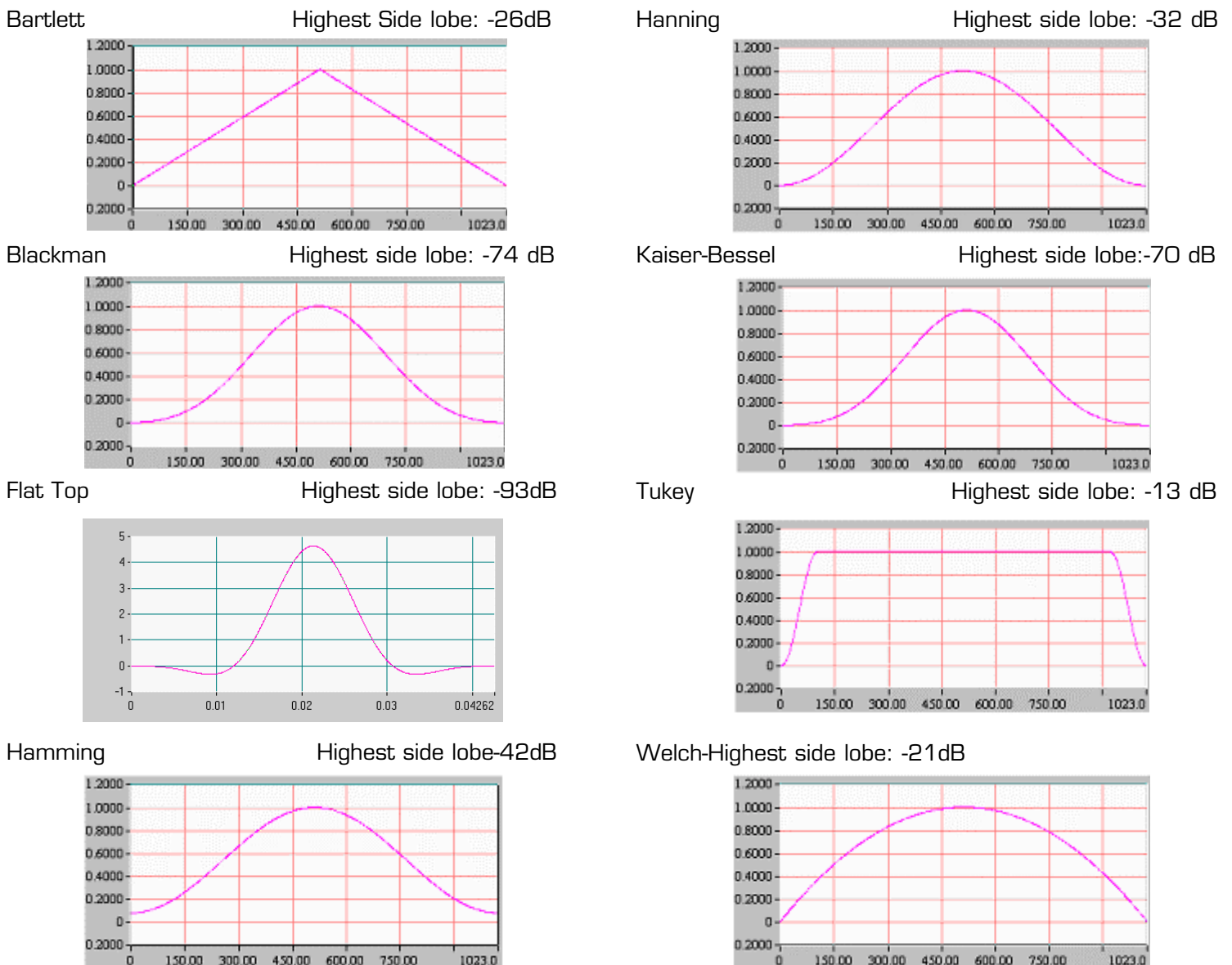*Figure 8. Frequency response function of a beam with and without window.*

Choosing a Windowing Function

FFT windows reduce the effects of leakage but can not eliminate leakage entirely. In effect, they only change the shape of the leakage. In addition, each type of window affects the spectrum in a slightly different way. Many different windows have been proposed over time, each with its own advantage and disadvantage relative to the others. Some are more effective for specific types of signal types such as random or sinusoidal. Some improve the frequency resolution, that is, they make it easier to detect the exact frequency of a peak in the spectrum. Some improve the amplitude accuracy, that is, they most accurately indicate the level of the peak. The best type of window should be chosen for each specific application.

The most common windows and their features are given below. This table can be used to choose the best windowing function for each application.

Window	Best for these Signal Types	Frequency Resolution	Spectral Leakage	Amplitude Accuracy
Barlett	Random	Good	Fair	Fair
Blackman	Random or mixed	Poor	Best	Good
Flat top	Sinusoids	Poor	**Good**	Best
Hanning	Random	Good	Good	Fair
Hamming	Random	Good	Fair	Fair
Kaiser-Bessel	Random	Fair	Good	Good
None (boxcar)	Transient & Synchronous Sampling	Best	Poor	Poor
Tukey	Random	Good	Poor	Poor
Welch	Random	Good	Good	Fair

Time Domain Window Shapes

Bartlett   Highest Side lobe: -26dB

Hanning   Highest side lobe: -32 dB

Blackman   Highest side lobe: -74 dB

Kaiser-Bessel   Highest side lobe:-70 dB

Flat Top   Highest side lobe: -93dB

Tukey   Highest side lobe: -13 dB

Hamming   Highest side lobe-42dB

Welch-Highest side lobe: -21dB

Overlap Processing

One of the disadvantages of windowing functions is that the beginning and end of the signal is attenuated in the calculation of the spectrum. This means that more averages must be taken to get a good statistical representation of the spectrum, increasing the time to complete the measurement. Overlap processing is a feature that is available in most signal analyzers that can recover the lost data and reduce the measurement time. This processing reduces the total measurement time by recovering a portion of each previous frame that otherwise is lost due to the effect of the windowing function as shown in Figure 9. The top pane shows the original continuous input signal. Below the Input Signal are shown the overlapping windowed frames. Next are the unaveraged FFTs from each frame, and finally at the bottom is the average of the FFTs. Overlap processing is particularly effective at reducing the measurement time for low frequency tests (generally under 50 Hz) for which the frame acquisition times are very long.
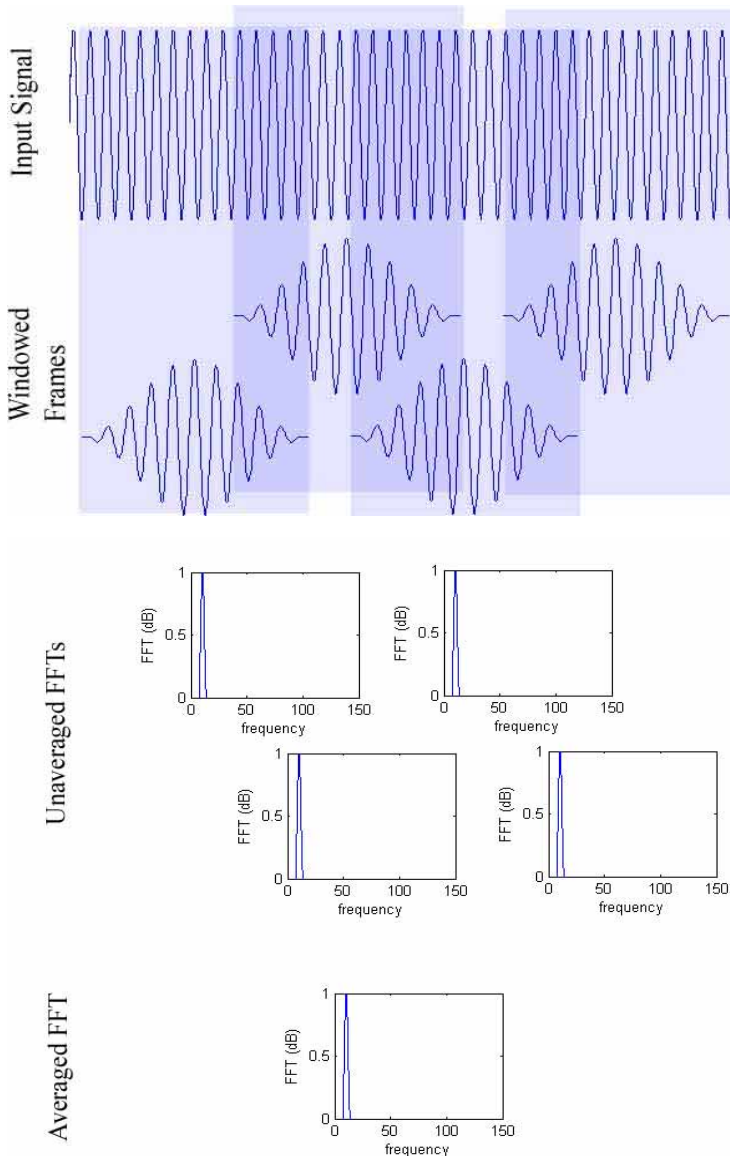

Figure 9. Overlap processing shortens the acquisition time by recovering a portion of each previous frame that otherwise is lost due to the effect of the FFT window

Windowing for Impact Measurements

Another type of windowing, developed especially for modal analysis using an impact hammer, is the exponential window. This window function, shown in Figure 10, has two parts, the pre-window at the beginning of the time frame, and the exponential window. The pre-window includes a hold-off period that eliminates any noise before the impact. The length of this hold-off period can be specified by the user to coincide with the pre-trigger time reducing the effects of noise. The exponential window applies an exponential decay that forces the response data to zero by the end of the frame resulting in a guaranteed periodic signal. It should be noted that this will result in an over estimate of the damping of the structure because the windowing function artificially damps the signal in a shorter time.
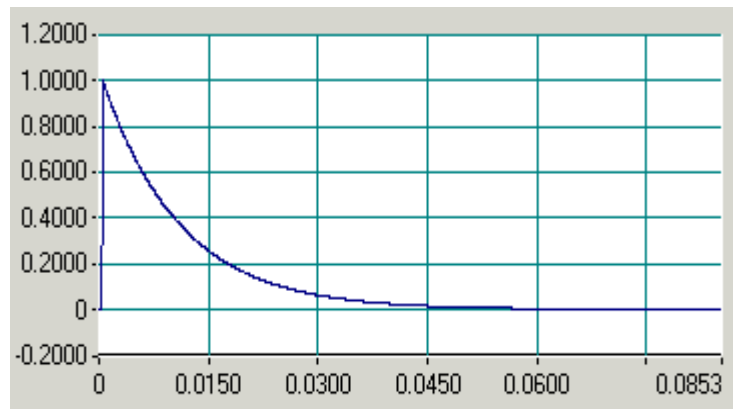

Figure 10. Force/exponential widow function is used for modal analysis with impact hammer excitation.

Older signal analyzer hardware was limited in memory and computational resources, limiting the data record to as few as 256 points. This limitation made exponential windowing necessary. Modern signal analyzers contain more memory and computational resources. When possible, it is always preferable to use no window by increasing the record length to capture the entire waveform in one time record.

Examples

The following examples describe three applications and the outline the typical decision making process on selecting the appropriate windowing function.

Case 1. A structure is excited with a mechanical shaker using broadband random noise and an accelerometer is placed on the structure to identify the resonant frequencies as accurately as possible. According to Table 1, Hanning, Hamming, Tukey and Welch produce good frequency resolution. Hanning is the most commonly used window function for random signals because it provides good frequency resolution and leakage protection with fair amplitude accuracy. Figure 11 shows a comparison of the spectrum with and without a window. The window reduces the leakage and provides more accurate amplitude measurements for the resonant frequencies.
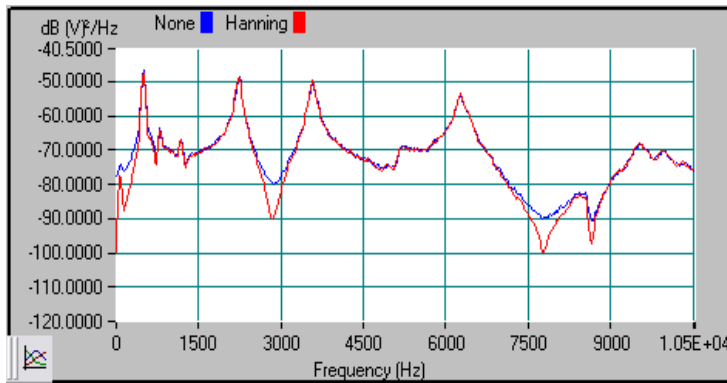

Figure 11. Spectrum with and without window.

Case 2. The same structure in Case 1 is next subjected to a pure sine tone at the first resonance frequency to accurately measure the amplitude ratio between the excitation and the structural response level. According to Table 1, a Flat Top window function gives the best amplitude accuracy for sinusoidal signals. Figure 12 shows the frequency response function with the Flat Top window. Note that the Flat Top window changes the shape of the peak from a sharp peak to a flat peak, as suggested by the name.
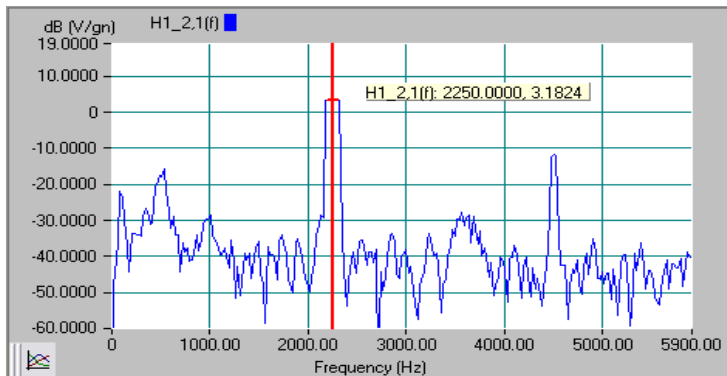

Figure 12. Frequency response function using Flat Top window.

Case 3. A measurement is performed on a lightly damped structure using an impact hammer. A few test impacts show that the structure continues to vibrate after the impact for a time of approximately 1.1 seconds. The time record on the signal analyzer is set to 800 milliseconds and therefore an exponential window is used. Figure 13 shows the time response of the structure without the window in the top frame. Note that the vibration has not died out at the end of the time record. The bottom frame shows the results with the window applied. The vibrations are forced to zero at the end of the time record by the exponential window.
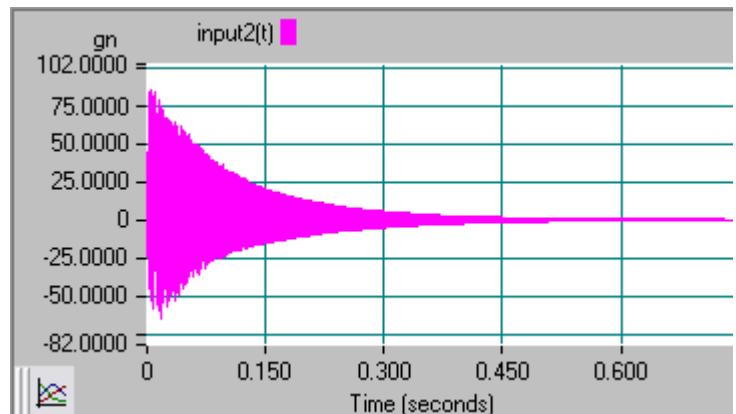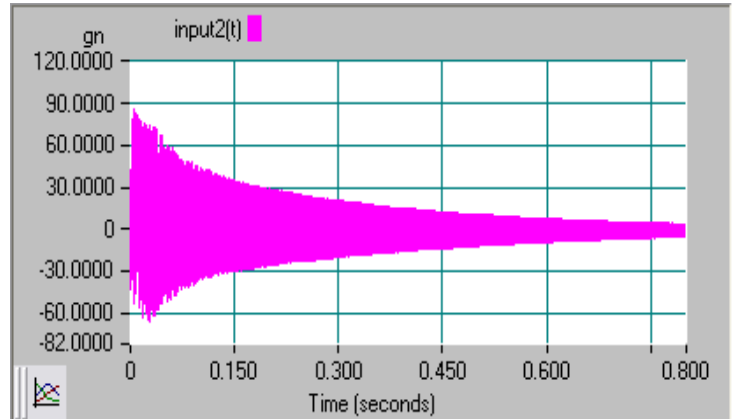

Figure 13. Time response of lightly damped structure without exponential window (top) and with window (bottom).

The plots in figure 13 show the disadvantages of using an exponential window. Estimates of damping extracted from the resulting frequency response measurements will be affected. This effect results from the exponential window adds artificial damping to the measurements.

Conclusions

All FFT based measurements assume that the signal is periodic in the time frame. When the measured signal is not periodic then leakage occurs. Leakage results in misleading information about the spectral amplitude and frequency. An FFT window can be applied to reduce the effects of leakage. There are many windows to choose from, each with advantages for specific applications. You must understand the effects of leakage and know the tradeoffs and advantages of the various windowing functions to accurately interpret frequency domain measurements.

References

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